Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel GCE Mathematics

Pure 1 Paper 9MA0/01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

October 2020
Publications Code 9MA0_01_2010_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of *x*. Do not accept ${}^{n}C_{r}$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^2+32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x, -8x^2, 32x^3$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or $\approx$.

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}} = 1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^2+32x^3$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\rightarrow\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

Question	Scheme	Marks	AOs
2	$4^{3p-1}=5^{210}\Rightarrow(3p-1)\log 4=210\log 5$	M1	1.1b
	$\Rightarrow 3p=\dfrac{210\log 5}{\log 4}+1\Rightarrow p=...$	dM1	2.1
	$p=$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.

Look for $(3p-1)\log 4=210\log 5\Rightarrow 3p=\dfrac{210\log 5}{\log 4}\pm 1\Rightarrow p=...$ condoning slips.

You may see numerical versions E.g. $(3p-1)\times 0.60=210\times 0.7\Rightarrow 1.8p-0.6=147\Rightarrow p=82$

Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4=210\log 5\Rightarrow 3p=210\log\dfrac{5}{4}\pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

....

There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210}\Rightarrow\dfrac{4^{3p}}{4}=5^{210}$

but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

....

Using base 4 or 5

E.g. $4^{3p-1}=5^{210}\Rightarrow(3p-1)=\log_4 5^{210}$

The M mark is not scored until $(3p-1)=210\log_4 5$

....

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as *OC* is parallel to *AB*, so *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why *OABC* is a trapezium.
Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$
It requires a reason and minimal conclusion.
Example 1:
$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium
Example 2:
A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.
Example 3
As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, *OC* and *AB* are parallel, so proven
Example 4
Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $\mathrm{f}^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $\mathrm{ff}(x)=\dfrac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\dfrac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\dfrac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $\mathrm{f}^{-1}(x)$. FYI $\mathrm{f}^{-1}(x)=\dfrac{2x-7}{x-3}$

The method for finding $\mathrm{f}^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\dfrac{3x-7}{x-2}$ into $\mathrm{f}(x)$. Condone slips but the expression must have a correct form. E.g. $\dfrac{3\times\left(\frac{*-*}{*-*}\right)-a}{\left(\frac{*-*}{*-*}\right)-b}$ where *a* and *b* are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\dfrac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\dfrac{P(x)}{Q(x)}\times\dfrac{x-2}{x-2}$

A1: Reaches $\dfrac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\dfrac{3x-7}{x-2}\equiv a+\dfrac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\dfrac{3x-7}{x-2}\equiv 3-\dfrac{1}{x-2}$

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2\times"17.4" = ...$	M1	3.4
	= 62.8 (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28\times"1.3265^4" = ...$ or $\dfrac{115}{"1.3265"}$	M1	3.4
	= 86.7 (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a+(n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\dfrac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2"d"$ or equivalent. This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\dfrac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\dfrac{115}{28}}$ condoning slips.

It is implied by stating or using $r = \text{awrt } 1.33$

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28\times"r^4"$ or $\dfrac{115}{"r"}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28\times"r^2"$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2} \Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha =$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C Remember to isw

Condone a lack of units. Follow through on their value of *R* so allow $(5+"R")$ °C

(c)

M1: An complete strategy to find *t* from $\frac{\pi t}{12} \pm 1.107 - 3 = \frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t \pm 63.4 - 171.9 = 90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

....

It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt} = \frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right) - \frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right) = 0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right) = \frac{1}{2} \Rightarrow t = 13.23 = 13{:}14$ scores M1 A1 A1

$\frac{d\theta}{dt} = \cos\left(\frac{\pi t}{12}-3\right) - 2\sin\left(\frac{\pi t}{12}-3\right) = 0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right) = \frac{1}{2} \Rightarrow t = 13.23 = 13{:}14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

....

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds *m*	M1	1.1b
	Equation of *l* is $y = 6x + 25$	A1	1.1b
	Attempts equation of *C* Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find *a*	M1	3.1a
	Equation of *C* is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region *R* is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)
Notes:			

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line *l*.

E.g. Attempt using two points: Finds $m = \pm\dfrac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find *c*. Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form *y* = ... but the correct equation can be implied from an inequality. E.g. $.... < y < 6x + 25$

.

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant *a* in $y = a(x \pm 2)^2 + 13$ or the constants *a*, *b* in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in *a* and *b* using the point (−2, 13) the gradient being 0 at *x* = −2. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25\ (2a - b = -6)$ and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $f(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25\, dx$

B1ft: Fully defines the region *R*. Follow through on their equations for *l* and *C*.

Allow strict or non -strict inequalities as long as they are used consistently.

E.g. Allow for example "$3(x+2)^2 + 13 < y < 6x + 25$" "$3(x+2)^2 + 13 \leqslant y \leqslant 6x + 25$"

Allow the inequalities to be given separately, e.g. $y < 6x + 25, y > 3(x+2)^2 + 13$. Set notation may be used so $\left\{(x,y) : y > 3(x+2)^2 + 13\right\} \cap \left\{(x,y) : y < 6x + 25\right\}$ is fine but condone with or without any of $(x,y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include "$y < 6x + 25$ **or** $y > 3(x+2)^2 + 13$", $\left\{(x,y) : y > 3(x+2)^2 + 13\right\} \cup \left\{(x,y) : y < 6x + 25\right\}$

Values of *x* could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25, x < 0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

....

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

....

Question	Scheme	Marks	AOs
9(a)	$\mathrm{f}(x)=4\left(x^2-2\right)\mathrm{e}^{-2x}$		
	Differentiates to $\mathrm{e}^{-2x}\times 8x+4\left(x^2-2\right)\times -2\mathrm{e}^{-2x}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=8\mathrm{e}^{-2x}\left\{x-\left(x^2-2\right)\right\}=8\left(2+x-x^2\right)\mathrm{e}^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $\mathrm{f}'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $\left(-1,-4\mathrm{e}^2\right)$ and $\left(2,8\mathrm{e}^{-4}\right)$	A1	1.1b
		(3)	
(c)	(i) Range $\left[-8\mathrm{e}^2,\infty\right)$ o.e. such as $\mathrm{g}(x)\geqslant -8\mathrm{e}^2$	B1ft	2.5
	(ii) For • Either attempting to find $2\mathrm{f}(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times$"$8\mathrm{e}^{-4}$"-3 and identifying this as the **upper bound**	M1	3.1a
	Range $\left[-19,16\mathrm{e}^{-4}-3\right]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $\mathrm{e}^{-2x}\to k\mathrm{e}^{-2x},\quad k\neq 0$

If candidate states $u=4(x^2-2), v=\mathrm{e}^{-2x}$ with $u'=....,v'=...\mathrm{e}^{-2x}$ it can be implied by their $vu'+uv'$

If they just write down an answer without working award for $\mathrm{f}'(x)=px\mathrm{e}^{-2x}\pm q\left(x^2-2\right)\mathrm{e}^{-2x}$

They may multiply out first $\mathrm{f}(x)=4x^2\mathrm{e}^{-2x}-8\mathrm{e}^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $\mathrm{f}(x)=\dfrac{u}{v}=\dfrac{4\left(x^2-2\right)}{\mathrm{e}^{2x}}$ with $v'=k\mathrm{e}^{2x}$ and $\mathrm{f}'(x)=\dfrac{vu'-uv'}{v^2}$

A1: A correct $\mathrm{f}'(x)$ which may be unsimplified.

Via the quotient rule you can award for $\mathrm{f}'(x)=\dfrac{8x\mathrm{e}^{2x}-8\left(x^2-2\right)\mathrm{e}^{2x}}{\mathrm{e}^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $\mathrm{f}'(x)$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $\mathrm{f}'(x)=8x\mathrm{e}^{-2x}-8\left(x^2-2\right)\mathrm{e}^{-2x}$

to the given answer in one step.

Do not allow it from an **unsimplified** $f'(x) = 4\times 2xe^{-2x} + 4\left(x^2-2\right)\times -2e^{-2x}$

Allow the expression / bracketed expression to be written in a different order.

So, for example, $8\left(x - x^2 + 2\right)e^{-2x}$ is OK

(b)

B1: States or implies $x = -1, 2$ (as the roots of $f'(x) = 0$)

M1: Substitutes one *x* value of their solution to $f'(x) = 0$ in f(*x*) to find a *y* value.

Allow decimals here (3sf). FYI, to 3 sf, $-4e^2 = -29.6$ and $8e^{-4} = 0.147$

Some candidates just write down the *x* coordinates but then go on in part (c) to find the ranges using the *y* coordinates. Allow this mark to be scored from work in part (c)

A1: Obtains $\left(-1, -4e^2\right)$ and $\left(2, 8e^{-4}\right)$ as the stationary points. This must be scored in (b). Remember to isw after a correct answer. Allow these to be written separately. E.g. $x = -1,\ y = -4e^2$

Extra solutions, e.g. from $x = 0$ will be penalised on this mark.

(c)(i)

B1ft: For a correct range written using correct notation.

Follow through on 2 × their minimum "*y*" value from part (b), providing it is negative.

Condone a decimal answer if this is consistent with their answer in (b) to 3sf or better.

Examples of correct responses are $\left[-8e^2, \infty\right)$, $g \geqslant -8e^2$, $y \geqslant -8e^2$, $\left\{q \in \mathbb{R}, q \geqslant -8e^2\right\}$

(c)(ii)

M1: See main scheme. Follow through on $2\times$ their "$8e^{-4}$" -3 for the upper bound.

A1: Range $\left[-19, 16e^{-4} - 3\right]$ o.e. such as $-19 \leqslant y \leqslant 16e^{-4} - 3$ but must be exact

Question	Scheme	Marks	AOs
10 (a)	$x=u^2+1 \Rightarrow \mathrm{d}x=2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int \frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)}=\int \frac{3\times 2u\,\mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p=2, q=3$	B1	1.1b
	$=\int \frac{3\times 2\cancel{u}\,\mathrm{d}u}{u^{\cancel{2}}(3+2u)}=\int \frac{6\,\mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)}=\frac{A}{u}+\frac{B}{3+2u} \Rightarrow A=..., B=...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6\,\mathrm{d}u}{u(3+2u)}=2\ln u-2\ln(3+2u) \qquad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u="3", u="2"$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3-2\ln 9)-(2\ln 2-2\ln 7)=2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
			(10 marks)
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $\mathrm{d}x=2u\mathrm{d}u$ or exact equivalent. E.g. $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$, $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x=u^2+1$, including $\mathrm{d}x \to ...u\mathrm{d}u$ to form an integrand in terms of u. Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p=2, q=3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B.

A1: Correct PF $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$ (Not scored for just the correct values of A and B)

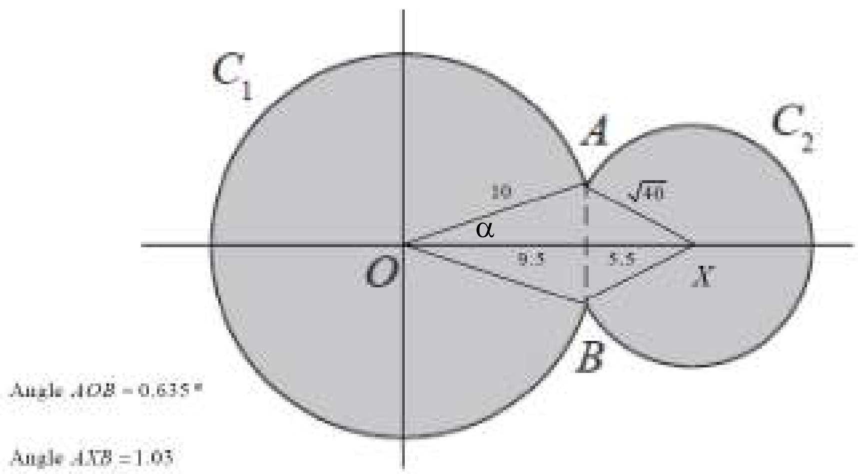
dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns. Look for $P\ln u+Q\ln(3+2u)$

A1ft: Correct integration for their $\frac{A}{u}+\frac{B}{3+2u} \to A\ln u+\frac{B}{2}\ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k\ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u's back to x's and use limits of 5 and 10.

A1: Proceeds to $\ln\frac{49}{36}$. Answers without working please send to review.

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle *AOB* in circle C_1 Eg Attempts $\cos\alpha=\dfrac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle *AOB* = $2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ = 56.48	M1	1.1b
	Attempts to find angle *AXB* or *AXO* in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-"9.5"}{\sqrt{40}}\Rightarrow\beta=...$ (Note *AXB* =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	= 89.7	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle *AOB* in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-\left(\sqrt{39}\right)^2}{2\times 10\times 10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)
M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}}+2\pi\sqrt{40}\underline{\underline{-10\times 0.635}}$... which is fine for this M1

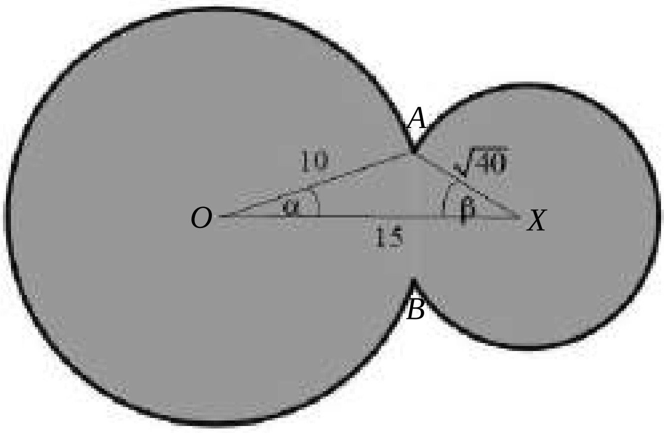
M1: Attempts to use a correct method in order to find angle *AXB* or *AXO* in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-\left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.
A1: awrt 89.7

....



(a)
M1: For the key step in attempting to find all lengths in triangle *OAX*, condoning slips
A1: All three lengths correct
M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

....

Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x-50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x-50°)$		
	$\cot x = \cot(3x-50°) \Rightarrow x = 3x-50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x-50°) \Rightarrow x+180° = 3x-50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in *x* (or other variable) instead of $\theta's$ here

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone $\theta's$ instead of *x*'s here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x-50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x+180° = 3x-50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x-50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

....

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x-50°) \Rightarrow \cot x = \cot(3x-50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{1}{\sin\theta} - \sin\theta = \operatorname{cosec}\theta - \sin\theta$ *	A1*	2.1
		(3)	

Alt 2- Works on both sides

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	1.2
	$LHS = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone = for $\equiv$)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x-50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x-50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \dfrac{k\left(a_n + 2\right)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$\left(a_1 = 2\right),\ a_2 = 2k,\ a_3 = k+1,\ a_4 = \dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2,\ a_{2/5} = -4,\ a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times \left(2 + -4 + -1\right) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \dfrac{k\left(a_n + 2\right)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \dfrac{k\left(2+2\right)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2,\ a_2 = 2k,\ a_3 = k+1, a_4 = \dfrac{k(k+3)}{k+1}$ and so $2 = \dfrac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4,\ a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\displaystyle\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26\times\left(2+-4+-1\right)+2+-4$

or $\displaystyle\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27\times\left(2+-4+-1\right)-\left(-1\right)$

For candidates who find in terms of *k* award for $27\times 2 + 27\times\left(2k\right) + 26\times\left(k+1\right)$ or $80k+80$

If candidates proceed and substitute $k=-2$ into $80k+80$ to get -80 then all 3 marks are scored.

A1: -80

....

Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3}\times\left(2+-4+-1\right) = 26\frac{2}{3}\times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

....

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ (for positive constant c)	B1	3.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ and $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	M1	2.1
	$-c=4\pi r^2\times\frac{\mathrm{d}r}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{c}{4\pi r^2}=-\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{k}{r^2}\Rightarrow\int r^2\mathrm{d}r=\int -k\mathrm{d}t$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3}=-kt(+\alpha)$	A1	1.1b
	Uses $t=0, r=40\Rightarrow\alpha=...$ $\qquad\alpha=\frac{64000}{3}$	M1	1.1b
	Uses $t=5, r=20\ \&\ \alpha=...\Rightarrow k=...$	M1	3.4
	$r^3=64000-11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. $"64000-11200t"\ ...\ 0\Rightarrow t...$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
			(10 marks)
Notes:			

(a)

B1: Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c).

Any "letter" is acceptable here including k.

Note that $\frac{dV}{dt} = c$ is B0 unless they state that c is a negative constant.

M1: For an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = 4\pi r^2$

Allow for an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = \lambda r^2$ (Any constant is fine)

There is no requirement to use the correct formula for the volume of a sphere for this mark.

A1*: Proceeds to the given answer with an intermediate line equivalent to $\frac{dr}{dt} = -\frac{c}{4\pi r^2}$

If candidate started with $\frac{dV}{dt} = -k$ they must provide a minimal explanation how

$\frac{dr}{dt} = -\frac{k}{4\pi r^2} \rightarrow \frac{dr}{dt} = -\frac{k}{r^2}$. E.g $\frac{1}{4\pi}$ is a constant so replace $\frac{k}{4\pi}$ with k

It is not necessary to use the full formula for the volume of a sphere, eg allow $V = \kappa r^3$ but if it has been quoted it must be correct. So using $V = 4r^3$ can potentially score 2 of the 3 marks.

(b)

M1: For the key step of separating the variables correctly AND integrating one side with at least one index correct. The integral signs do not need to be seen.

A1: Correct integration E.g. $\frac{r^3}{3} = -kt(+\alpha)$ or equivalent. The $+\alpha$ is not required for this mark.

This may be awarded if k has been given a value.

M1: Uses the initial conditions to find a value for the constant of integration α

If a constant of integration is not present, or k has been given a pre defined value, then only the first two marks can be awarded in part (b)

The mark may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

M1: Uses the second set of conditions with their value of α to find k

This may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

A1: Obtains any correct equation for the model.

E.g. $r^3 = 64000 - 11200t$ or exact equivalent such as $\frac{r^3}{3} = \frac{64000}{3} - \frac{11200}{3}t$.

ISW after sight of a correct answer. Condone recurring decimals e.g. $21333.\dot{3}$ for $\frac{64000}{3}$

Do not award if **only the** rounded/truncated decimal equivalents to say $\frac{64000}{3}$ is used.

(c)

M1: Recognises that the model is only valid when $r \geqslant 0$ **and uses this to find t.** Condone $r > 0$

Award for an attempt to find the value of t when $r = 0$. See scheme.

It must be from an equation of the form $ar^n = b - ct, \quad a,b,c > 0$ which give + ve values of t.

A1ft: Allow valid for times up to (and including) $\frac{40}{7}$ seconds, 5.71 seconds. Allow $t < \frac{40}{7}$ or $t \leqslant \frac{40}{7}$

There is no requirement for the left hand side of the inequality, 0

States invalid for times greater than $\frac{40}{7}$ seconds, 5.71 seconds.

Follow through on their equation so allow $t <$ their "$\frac{64000}{11200}$" as long as this value is greater than 5

($t = 5$ is one of the values in the question)

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2 \sec^2 y \dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x\times\dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18\times(x^4+81)-(-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y \dfrac{dy}{dx}$ or $\cot y \to -\operatorname{cosec}^2 y \dfrac{dy}{dx}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y \dfrac{dy}{dx} = ...$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x\tan y + x^2\sec^2 y \dfrac{dy}{dx} = 0$

Allow also $\sec^2 y \dfrac{dy}{dx} = -\dfrac{18}{x^3}$ or $2x = -9\operatorname{cosec}^2 y \dfrac{dy}{dx}$ amongst others

M1: Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$

(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu'-uv'}{v^2}$ with $u=-18x, v=x^4+81, u'=\pm18, v'=...x^3$

If no method is seen or implied award for $\frac{\pm18\times\left(x^4+81\right)\pm18x\left(ax^3\right)}{\left(x^4+81\right)^2}$

Using the product rule award for $\pm18\left(x^4+81\right)^{-1}\pm18x\left(x^4+81\right)^{-2}\times cx^3$

A1: Correct **simplified** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{54\left(x^4-27\right)}{\left(x^4+81\right)^2}$ o.e. such as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{54x^4-1458}{\left(x^4+81\right)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0\Rightarrow x^4=27\Rightarrow x=\sqrt[4]{27}$

Or for substituting $x=\sqrt[4]{27}$ into an unsimplified but correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.
See scheme.

It can be also be argued from $x^4<27$, $x^4=27$ and $x^4>27$ provided the conclusion states that the point of inflection is at $x=\sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ and showing that $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\neq0$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ when $x=\sqrt[4]{27}$

FYI $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{23328x^3}{\left(x^4+81\right)^3}=0.219$ when $x=\sqrt[4]{27}$

Alternative part (a) using arctan

M1: Sets $y=\arctan\frac{9}{x^2}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times...$ where ... could be 1

A2: $y=\arctan\frac{9}{x^2}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times-\frac{18}{x^3}$

A1*: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\frac{81}{x^4}}\times-\frac{18}{x^3}=\frac{-18x}{x^4+1}$ showing correct intermediate step and no errors.

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers *p* and *q* such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers *p* and *q* such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for *p* and *q* to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive *p* and *q*.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either *p* or *q* to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p,q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2-q^2=25 \Rightarrow q^2=4p^2-25$ with * or ** Or There are positive integers p and q such that $4p^2-q^2=25 \Rightarrow 4p^2=q^2+25$ with * or **	M1	2.1
	Sets $q=2n\pm1$ and expands $(2n\pm1)^2=4p^2-25$	M1	2.2a
	Proceeds to an expression such as $4p^2=4n^2+4n+26=4\left(n^2+n+6\right)+2$ $4p^2=4n^2+4n+26=4\left(n^2+n\right)+\dfrac{13}{2}$ $p^2=n^2+n+\dfrac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2-q^2=4\times(2m)^2-\left(2n+1\right)^2=16m^2-4n^2-4n-1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2-q^2=4\times(2m+1)^2-\left(2n+1\right)^2=16m^2+16m-4n^2-4n+3$	Three more than a multiple of 4 so cannot equal 25

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom